TRINITY COLLEGE

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

If required by your examination administrator, please place your student identification label in this box

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): |

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (90 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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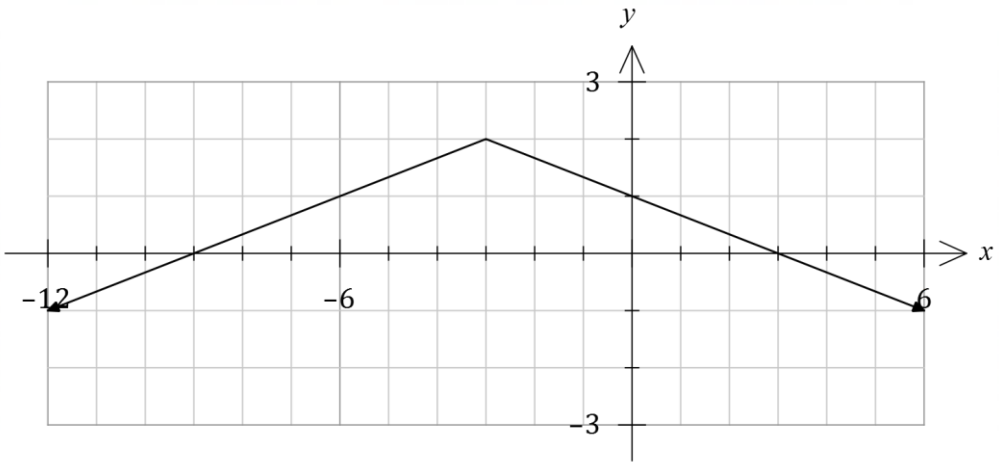
Question 8 **(8 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = a - |bx + c|$ and a, b and c are all positive constants.



(a) Determine the value of each of the constants a, b and c. (3 marks)

(b) Using the graph, or otherwise, solve

(i) $f(x) = 1$. (1 mark)

(ii) $f(x) = |x| - 3$. (2 marks)

(iii) $3f(x) = |x - 3|$. (2 marks)

See next page

Question 9 **(7 marks)**

Point C lies on a sphere with centre O, radius r and diameter AB.

(a) Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. Use a vector method to prove that AC is perpendicular to BC. (4 marks)

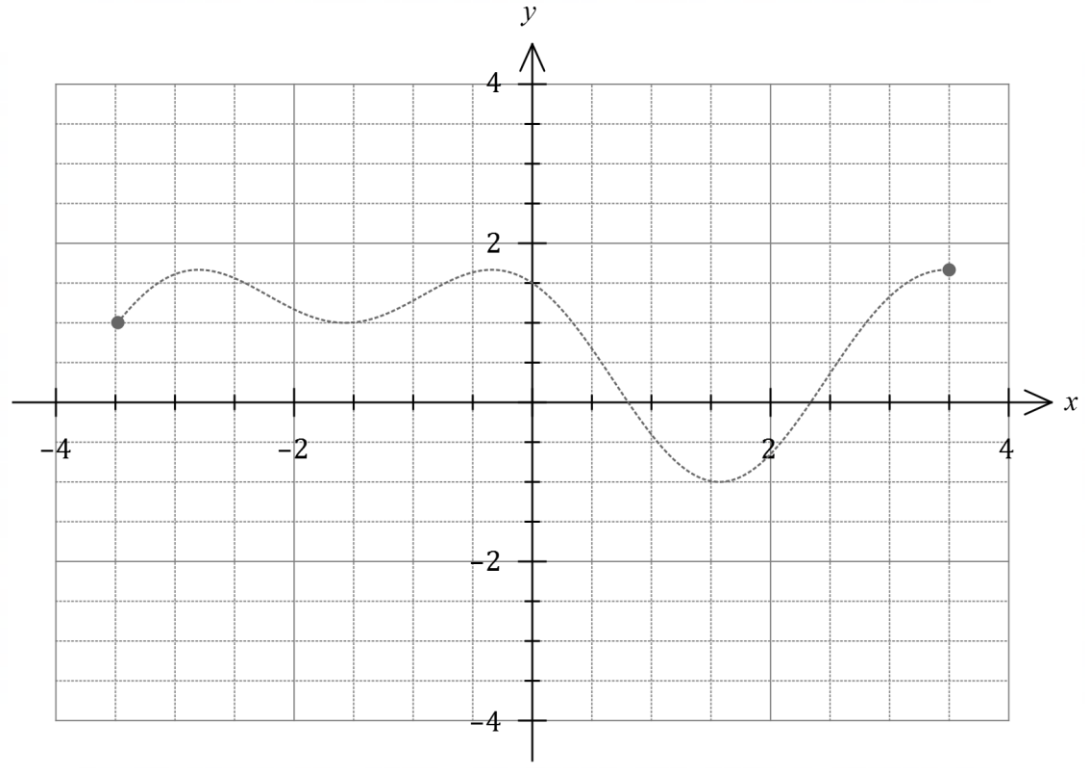
(b) If the position vectors of A, B and C are $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} k \\ -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$ respectively, determine the value of the constant k. (3 marks)

See next page

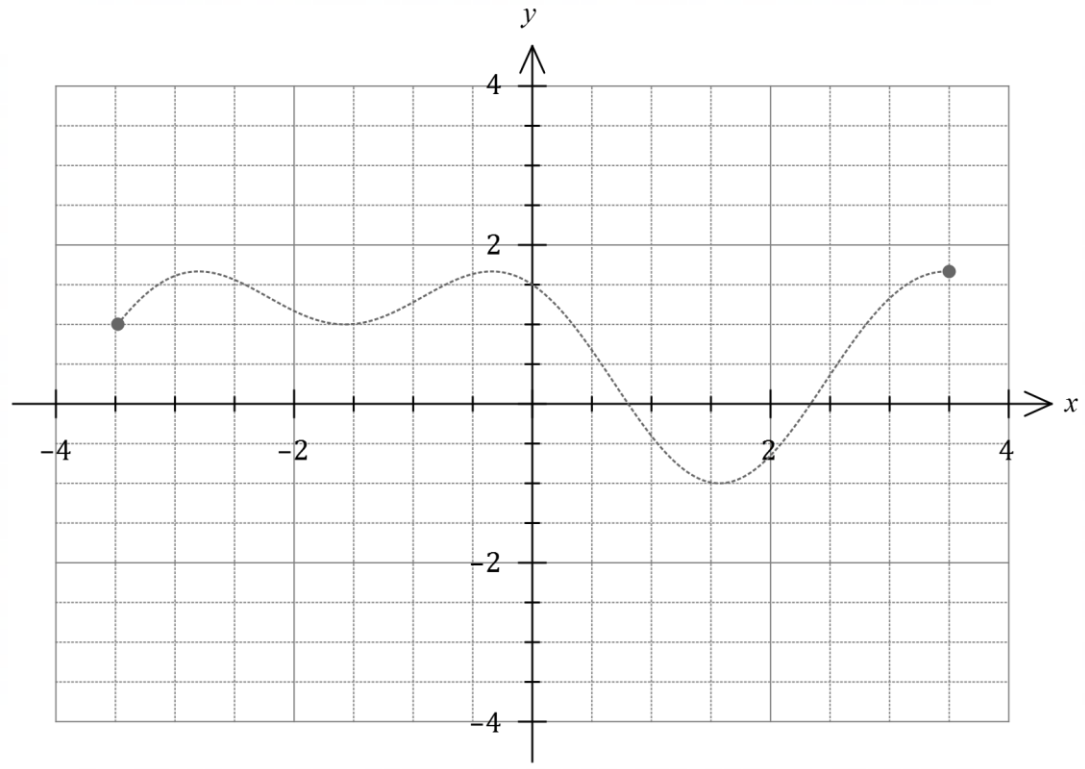
Question 10 **(8 marks)**

In each part of this question, the dotted curve shown is the graph of $y = f(x)$.

(a) Sketch the graph of $y = |f(x)|$. (2 marks)



(b) Sketch the graph of $y = \frac{1}{f(x)}$. (4 marks)



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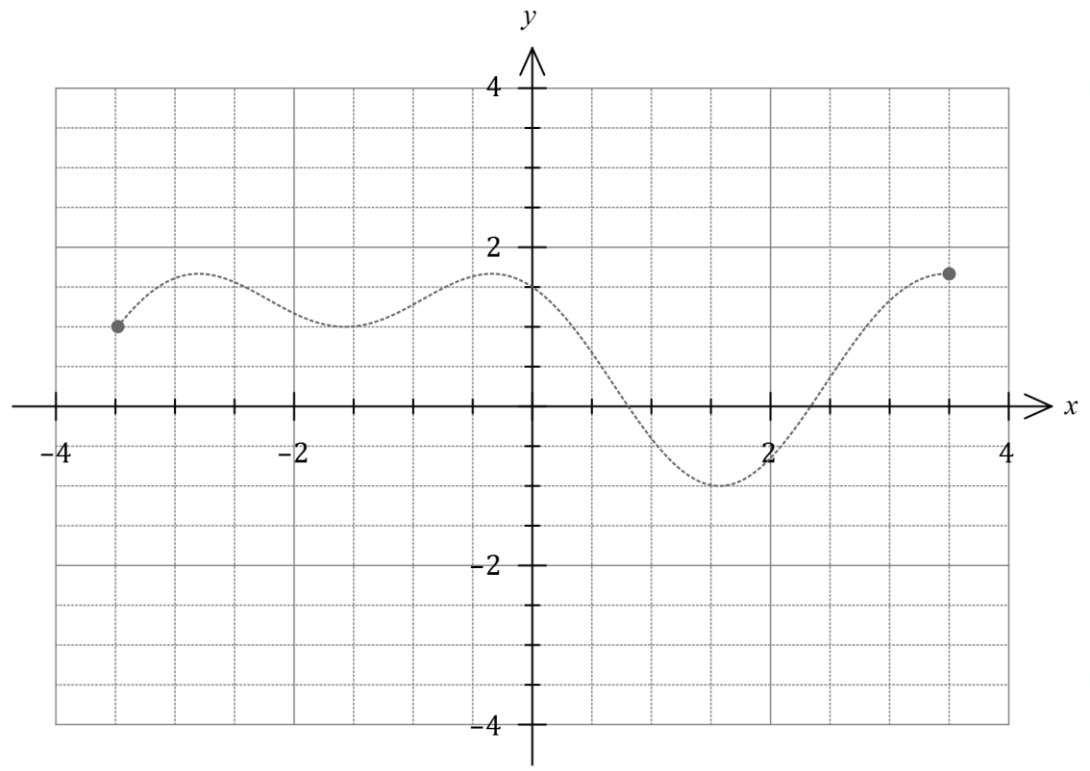
(c) Sketch the graph of $y = f(-|x|)$. (2 marks)



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Question 11 **(7 marks)**

(a) Solve the equation $32z^5 - i = 0$, giving exact solutions in the form $r\,\text{cis}\,\theta$, $-\pi < \theta \leq \pi$. (4 marks)

(b) One solution of the equation $z^n = 1$, where n is a positive integer, is $z = \text{cis}\left(\frac{11\pi}{13}\right)$.
If N solutions of the equation satisfy $-\frac{\pi}{4} < \arg(z) < 0$, determine, with reasoning, the least value of N. (3 marks)

See next page

Question 12 **(7 marks)**

A small body is moving with constant velocity in space so that initially it is located at $(-9, 3, 5)$ and three seconds later it is at $(-12, -6, 11)$, where all dimensions are in metres.

(a) Determine a vector equation for the position of the small body at time t seconds. (2 marks)

A laser beam shines along the line with equation $\frac{x-2}{-2} = \frac{y+4}{-2} = \frac{z+7}{3}$

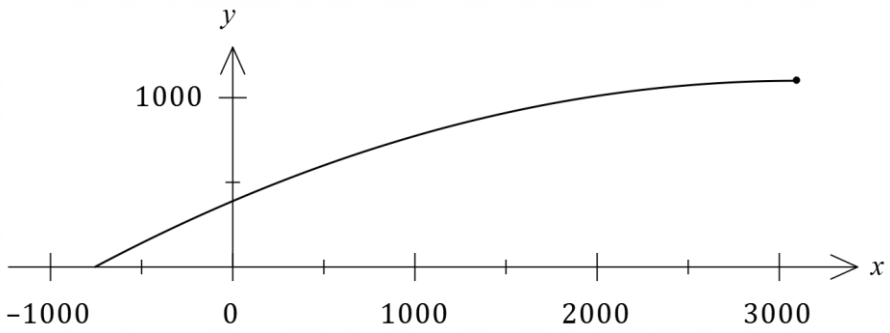
(b) Write the vector equation of this line in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. (1 mark)

(c) Show that the small body passes through the laser beam and state where this occurs. (4 marks)

See next page

Question 13 **(10 marks)**

An aeroplane flying at a constant altitude releases a bomb at $3100\mathbf{i} + 1100\mathbf{j}$ with an initial velocity of $-260\mathbf{i}$. The path of the bomb is shown below.



Assume there is no wind in the region, air resistance can be ignored and the only acceleration acting on the bomb is $-10\mathbf{j}$ ms^{-2} due to gravity.

(a) Use the acceleration vector of the bomb to clearly deduce that its position vector at time t seconds after release is $\mathbf{r}(t) = (3100 - 260t)\mathbf{i} + (1100 - 5t^2)\mathbf{j}$. (3 marks)

(b) Determine the speed of the bomb 7 seconds after it is released. (2 marks)

See next page

Four seconds after the bomb is released, a projectile is launched from the origin with a speed of v_0 at an angle of elevation of $\theta°$ to intercept it at a height of 600 m.

The position vector of the projectile T seconds after its launch is

$$\mathbf{r}(T) = (v_0 \cos(\theta)\, T)\mathbf{i} + (v_0 \sin(\theta)\, T - 5T^2)\mathbf{j}.$$

(c) Determine the value of v_0 and the value of θ so that the projectile intercepts the bomb.

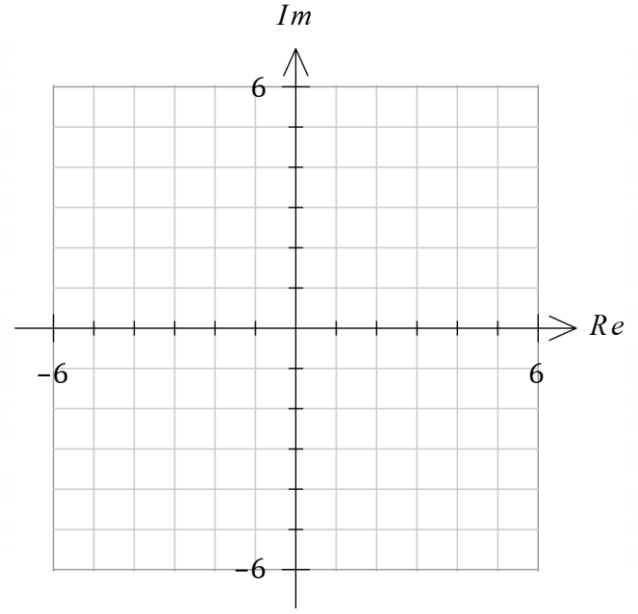
(5 marks)

See next page

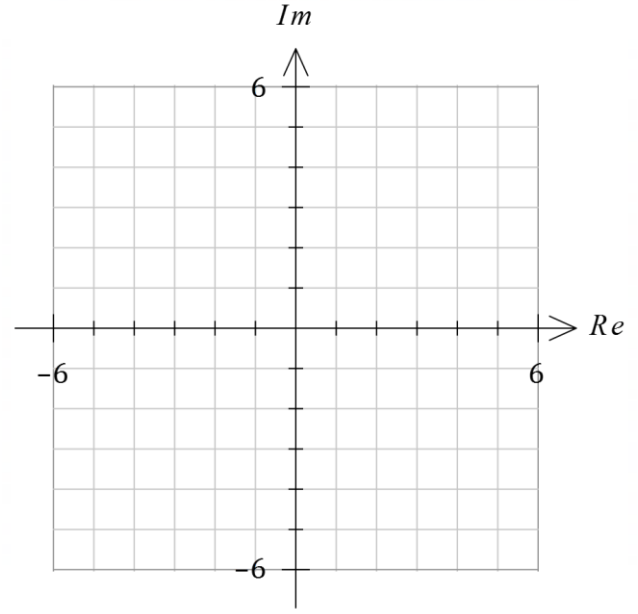
Question 14 **(8 marks)**

(a) On the Argand planes below sketch the locus of the complex number $z = x + iy$ given by

(i) $|z + 3 - 4i| = |z - 2 + i|$. (3 marks)



(ii) $|\bar{z} + 3i| \leq 3$. (3 marks)



(b) For the locus $|z + 3 - 4i| = |z - 2 + i|$ in part (a), determine the minimum value for $|z + 4i|$. (2 marks)

See next page

Question 15 **(6 marks)**

Points P, Q and R lie in plane Π with position vectors $\begin{pmatrix}-2\\8\\1\end{pmatrix}$, $\begin{pmatrix}-1\\7\\2\end{pmatrix}$ and $\begin{pmatrix}0\\5\\0\end{pmatrix}$ respectively.

(a) Determine the vector equation for plane Π in the form $\mathbf{r} \cdot \mathbf{n} = k$. (3 marks)

The equation of line L is $\mathbf{r} = \begin{pmatrix}2\\3\\-1\end{pmatrix} + \lambda\begin{pmatrix}-1\\3\\5\end{pmatrix}$.

(b) Determine, if possible, where line L intersects with plane Π. If not possible, explain why not. (3 marks)

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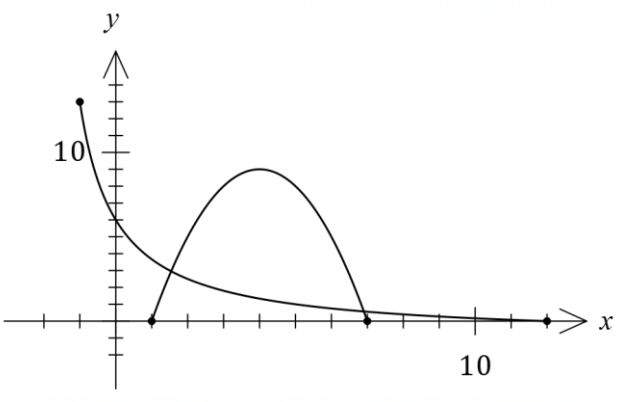
Question 16 **(8 marks)**

The graphs of $y = f(x)$ and $y = g(x)$ are shown at right.



The functions are defined by

$$f(x) = \frac{12 - x}{x + 2}, \qquad -1 \leq x \leq 12$$

and

$$g(x) = -x^2 + 8x - 7, \qquad 1 \leq x \leq 7.$$

(a) Explain why the inverse of g is not a function. (1 mark)

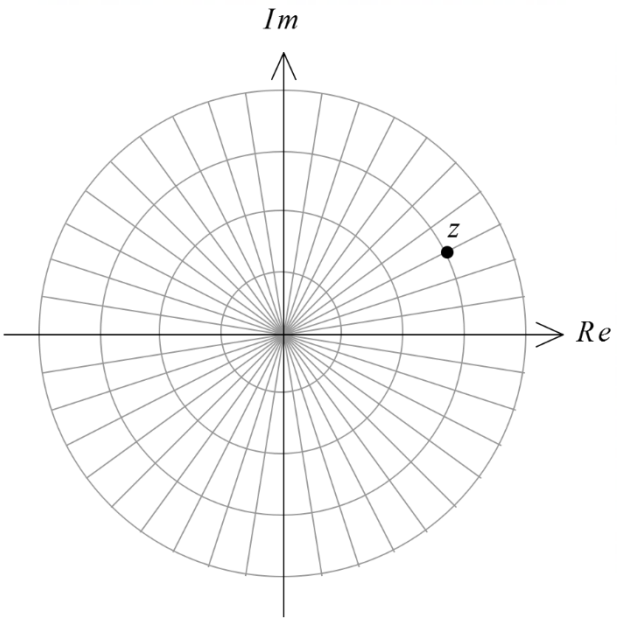
(b) Determine the definition for the inverse of f. (3 marks)

(c) Determine $g \circ f(0)$. (1 mark)

(d) Determine the domain for the function $g \circ f(x)$. (3 marks)

Question 17 **(5 marks)**

The complex number z is shown on the Argand diagram below and $w = \cos\left(-\frac{\pi}{20}\right) + i\sin\left(-\frac{\pi}{20}\right)$.



(a) Describe the geometric transformation performed by w when another complex number is multiplied by it, and plot and label zw on the Argand diagram. (2 marks)

(b) Plot and label the complex number zw^{2022} on the Argand diagram. (3 marks)

See next page

Question 18 **(8 marks)**

The vector equation of sphere S_1 is $|\mathbf{r} - (\mathbf{i} - 13\mathbf{j} + 10\mathbf{k})| = 18$. The position vector of the centre of sphere S_2 is $-2\mathbf{i} + 11\mathbf{j} - 2\mathbf{k}$ and the position vector of a point that lies on S_2 is $2\mathbf{i} + 12\mathbf{j} - 10\mathbf{k}$.

(a) Determine the Cartesian equation of sphere S_2. (2 marks)

(b) The equation of line L_1 is $\mathbf{r} = -4\mathbf{i} + 6\mathbf{j} + \lambda(-\mathbf{i} - \mathbf{j} + 4\mathbf{k})$. Show that L_1 is tangential to S_1 and determine the position vector for the point of tangency. (4 marks)

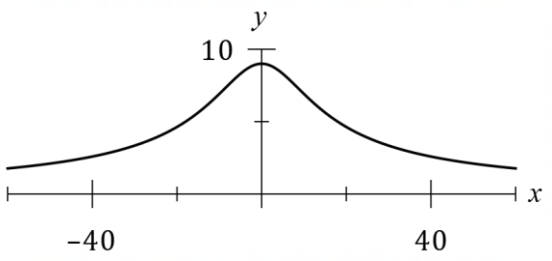
(c) Show that S_1 and S_2 are tangential. (2 marks)

See next page

Question 19 **(8 marks)**

The path of a small drone flying above level ground (the x-axis) is shown in the diagram, where $0 < t < 6\pi$. The position vector of the drone is

$$\mathbf{r}(t) = 12\cot\left(\frac{t}{6}\right)\mathbf{i} + 9\sin\left(\frac{t}{6}\right)\mathbf{j}\text{ m}.$$



(a) State, with reasoning, whether the drone is moving from left to right or from right to left. (2 marks)

(b) Determine the Cartesian equation for the path of the drone. (3 marks)

(c) Determine the distance travelled by the drone when its height above the ground is at least 4.5 metres, correct to the nearest centimetre. (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____